# Nordic-Baltic Physics Olympiad 2022

## 1. Escape (8 points) — *Päivo Simson.*

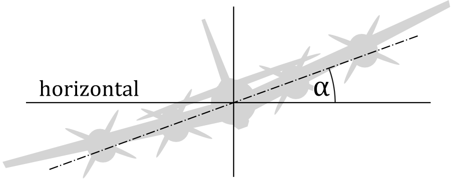
During a nuclear weapon test, a bomb is dropped from an airplane at an altitude of $H = 9\,\mathrm{km}$ and is set to detonate at $h = 500\,\mathrm{m}$ above the ground. The air drag acting on the bomb during the fall is negligible. Immediately after releasing the bomb, the plane starts to escape the explosion. The crew is protected from radiation burst by a protective screen, but the plane is vulnerable to the shockwave and needs to be as far as possible from the detonation point.

**i)** *(1 point)* The maximum speed of the airplane for a straight level flight (constant altitude) is $v_0$. What is the maximum diving angle so that the speed does not exceed the speed of sound $c$? The mass of the airplane is $m$, the air drag force is $F_d = kv^2$, and the gravitational acceleration is $g$.

For simplicity, assume from now on that the airplane stays at a constant altitude, flies at a constant speed $v = 190\,\mathrm{m/s}$, and all air maneuvers are limited by the maximum allowed lift-to-weight ratio $n = 2.5$. The gravitational acceleration $g = 9.81\,\mathrm{m/s^2}$.

**ii)** *(1 point)* After releasing the bomb, how much time does the airplane have before the radiation burst from the bomb hits it?

**iii)** *(1 point)* What is the smallest possible curvature radius $R$ of the plane's trajectory and the corresponding bank angle $\alpha$ (cf figure) of the plane?




**iv)** *(3 points)* Suggest a trajectory for the fastest escape. Calculate all the parameters that define the shape of the trajectory and its position relative to the detonation point.

**v)** *(2 points)* Based on the suggested trajectory, how far is the airplane from the detonation point when the shockwave hits it? It is estimated that the safe distance is 25 km. Is the plane able to escape the explosion? Assume that the average traveling speed of the shock wave is $u = 350\,\mathrm{m/s}$. For this part, if needed, you can use justified approximations to simplify the algebra.

## 2. Gas (6 points) — *Jaan Kalda.*

A box of volume $V$ contains $\nu$ moles of a monoatomic gas of molar mass $\mu$ at a negligibly low temperature. The box stops instantaneously after having moved with a constant speed $v$ (much greater than the thermal speed).

**i)** *(2 points)* Find the temperature inside the box after thermalization.

**ii)** *(2 points)* Find pressure to the front wall (the wall that was at the front while the box was moving) immediately after stopping.

**iii)** *(2 points)* Now, a spherical ball of gas (helium, $\mu = 4\,\mathrm{g/mol}$ of radius $r = 1\,\mathrm{cm}$, at temperature $T = 300\,\mathrm{K}$ is surrounded by vacuum. The mean free path of the molecules is much larger than $r$. At a certain moment, the walls of the ball break and after $\tau = 5\,\mathrm{ms}$, a part of the gas is trapped by erecting instantaneously walls forming a cubical vessel of volume $V = 1\,\mathrm{m^3}$. Find the temperature of the gas inside the cube $T$' after thermalization; neglect the thermal capacitance of the walls of the cubical vessel. $R = 8.31\,\mathrm{J/kg\,K}$.

## 3. Rocket (5 points) — *Jaan Kalda.*

A photon rocket is accelerated by a laser beam sent from the ground: the rocket's mirror reflects the photons in the exact opposite direction. The rocket's rest mass $M_0$ did not change during the ride. The total energy of the photons emitted by the laser (and later reflected by the rocket) is $\alpha M_0 c^2$. The laser power is constant over time.

**i)** *(1 point)* What speed $v$ does the rocket reach if $\alpha = 1 \times 10^{-6}$?

**ii)** *(2 points)* What speed $v$ does the rocket reach if $\alpha = 1$?

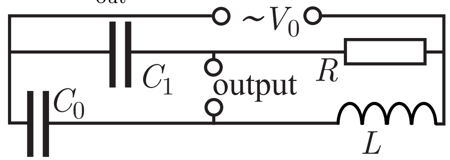
**iii)** *(2 points)* How many times does the rocket's acceleration, as perceived by passengers (i.e. the inertial force acting on them) differ at the beginning and at the end of the acceleration if $\alpha = 1$? Express the answer in terms of the rocket's final speed $v$.

## 4. AC filter (5 points) — *Jaan Kalda.*

An alternating voltage with amplitude $V_0$ and circular frequency $\omega_0$ is applied to the circuit shown below.

**i)** *(2 points)* For which circular frequency $\omega_0$ would the output voltage be infinite?

**ii)** *(3 points)* Now let the circular frequency $\omega$ be twice the value found in the previous task, $\omega = 2\omega_0$. The capacitance $C_1$ is chosen such that the phase difference $\varphi$ between the input and output voltages is maximum (parameters $C_0$, $L$ and $R$ are not changed). Find the phase shift $\varphi$ and the output voltage amplitude $V_{\mathrm{out}}$.




## 5. Ferromagnetic stripe (12 points) — *Jaan Kalda, Eero Uustalu.*

**Tools** A caliper, a ruler, graph papers, a resistive magnetic field sensor connected to batteries in a battery holder, a multimeter with two wires, a magnet a stripe made from soft ferromagnetic material of thickness $0.25\,\mathrm{mm}$ — **do not bend excessively to avoid damaging**.

**i)** *(0.5 points)* Connect the banana ends of the wires to the COM port and to the VΩmA-port of the multimeter. Switch on the multimeter in 20 volt (DC) range, and touch the two metallic leads of the battery holder (which are next to the points were the red and black wires come out from the holder) with the crocodile ends of the wires. Record the voltage $\mathcal{E}$ on the output leads of the battery holder. If the voltage is below $3.0\,\mathrm{V}$, you may ask for replacement batteries.

For all your magnetic field measurements, keep in mind that *if the battery voltage were to be exactly* $3\,\mathrm{V}$*, each millivolt in the reading would correspond to 10 microteslas of the magnetic field strength. However, the reading in millivolts is proportional to both the magnetic field and to the battery voltage.*

Connect the crocodiles to the yellow and red wires of the magnetic sensor. Keep in mind that (a) the sensor may have a non-zero offset: even if there is no magnetic field, the multimeter reading $V_0$ might be non-zero; (b) there is always the magnetic field of Earth. In what follows, avoid measuring magnetic fields which correspond to voltage readings bigger than $500\,\mathrm{mV}$ — such strong fields may cause changes in the offset value $V_0$. If you accidentally expose sensor to such fields, determine and use the new value of $V_0$.

The magnetic sensor has a small white dot marked on one of its edges. This points to the direction of that magnetic field component which is being measured.

**ii)** *(1.5 points)* Determine the offset voltage $V_0$ and the magnitude of the Earth's magnetic field $B_E \equiv |\vec{B}_E|$, and the angle between the vertical direction and the direction of $\vec{B}_E$.

Now, attach the magnet to the ferromagnetic stripe so that its circular face is touching the stripe's surface near one of its ends. Let us use a perpendicular system of coordinates where $x-y$-plane is the plane of the stripe, with the longest symmetry axis of the stripe serving as the $x$-axis, and $x = 0$ being at the position of the centre of the magnet.

The total magnetic field is the superposition of the field of the permanent magnet $\vec{B}_m$, the field of the magnetised ferromagnetic stripe $\vec{B}$, and the Earth's magnetic field $\vec{B}_E$. Below we are interested only in $\vec{B}$. Assume that $\vec{B}_m$ depends only on the distance from the magnet and remains unchanged when the magnet is detached from the stripe.

**iii)** *(2.5 points)* Measure the vertical field $B_z = B_z(L/2, y)$ caused by the stripe with the magnet, as a function of $y$, for $-w/2 \le y \le w/2$, at $x = L/2$, where $w$ denotes the width and $L$ — the length of the stripe. Find the ratio $\kappa = \langle B_z \rangle$ to $B_z(L/2, 0)$, where the average magnetic field

$$\langle B_z \rangle \equiv \int_{-w/2}^{w/2} B_z(L/2, y)\,\mathrm{d}y.$$

Assume that $\kappa$ remains constant along the stripe.

**iv)** *(3.5 points)* Measure $B_z(x, 0)$ near the surface of the stripe, as a function of $x$, and plot the measurement results.

**v)** *(2.5 points)* Let $J_s$ denote the saturation magnetisation of the stripe material; estimate the value of $J_s \mu_0$ (this is, roughly speaking, the strongest magnetic B-field which the ferromagnet is able to carry).

**vi)** *(1.5 points)* Prove experimentally that by small values of $x$, the magnetisation inside the stripe has reached saturation.

## 6. Life hacks (6 points) — *Jurij Bajc and Jaan Kalda.*

A normal healthy eye is able to see an object clearly if the object is located at a distance between 25.0 cm to infinity from the eye. A nearsighted eye sees equally well with the aid of a contact lens with an optical power of −6.00 dioptres.

**i)** *(1 point)* What is the clear-vision range for the nearsighted eye without the contact lens?

**ii)** *(2 points)* If a person uses glasses instead of contact lenses and the lens in the glasses is 2.00 cm from the eye, what is the adequate optical power of the lens to aid the nearsighted eye to see normally?

**iii)** *(3 points)* A front-wheel-drive car can hold steady on a sloping asphalt road with the brakes blocking all the four wheels when the road slope is no more than 45 degrees, and it can drive uphill when the road slope is no more than 22 degrees. What is the maximum slope of the road along which the car can reverse uphill? Assume that the centre of gravity of the car is equidistant from both the front and rear wheels.

## 7. Electrons in magnetic field (9 points) — *Kaarel Hänni, Jaan Kalda.*

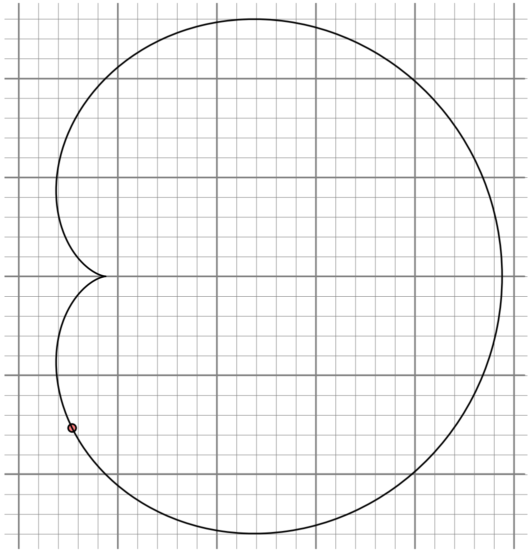
In what follows we consider two electrons (of mass $m$ and charge $-e$) moving in homogeneous magnetic field of flux density $B$ in such a way that the distance between the two electrons remains always constant; different subtasks explore different possibilities.

**i)** *(2 points)* The distance between the electrons is so large that their electrostatic repulsion can be neglected. At a certain moment, the angle between the velocity vectors of the electrons is $\alpha \neq 0$, and one of them moves in the direction of the other with speed $v$. Sketch the trajectories of the both electrons. What is the speed of the other electron?

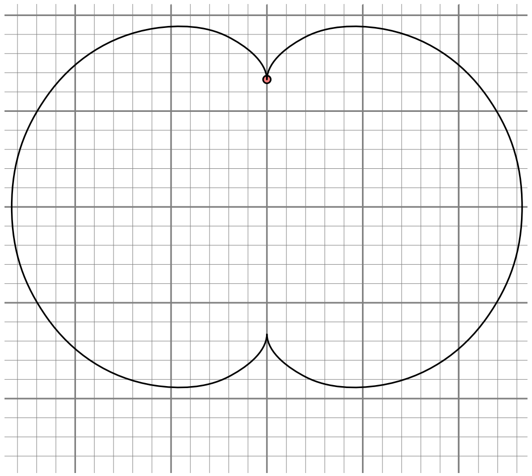
**ii)** *(1 point)* The electrostatic repulsion is still to be neglected. Now, the trajectories of the two electrons intersect, and at a certain moment, the velocity of one of them is $\vec{v}$. What can be said about the velocity of the other electron at the same moment? Sketch the both trajectories.

**iii)** *(2 points)* What is the distance $l$ between the electrons if both of them perform a periodic motion of period $\frac{6\pi m}{Be}$ while moving with a constant speed?

**iv)** *(2 points)* The figure below depicts the trajectory of one of the electrons for a certain set of initial conditions. Where is the other electron at the moment when the first electron is at the position of the small circle? What is the period of this motion?



**v)** *(2 points)* The figure below depicts the trajectory of one of the electrons for a certain set of initial conditions. What is the speed of the other electron at the moment when the first electron is at the position of the small circle?



## 8. Brightness of planets (9 points) — *Topi Löytäinen, Jaan Kalda.*

In this problem, the time periods are measured in years (y), and the distances are measured in astronomical units (au); the distance of Earth from Sun is $R_\oplus = 1\,\mathrm{au}$. The planets and Moon are assumed to move in the same plane, the ecliptic, around circular orbits in the same direction. Distance of Venus from Sun is $R_♀ = 0.72\,\mathrm{au}$, and distance of Mars from Sun is $R_♂ = 1.5\,\mathrm{au}$. In some parts below, you will need to assume that the planets are so-called Lambertian scatterers, i.e. when you can see the full disc of a planet illuminated by sunlight, middle parts of the disc seem exactly as bright as the parts at the edge of the disc. Throughout the problem, assume that the observer is situated in Tallinn, at the geographical coordinates 59.5°N and 24.7°E. The axis of Earth is tilted by 23.5°with respect to the normal of the ecliptic plane.

**i)** *(0.7 points)* When can you see a waxing crescent moon? Select one or more options (A, B and/or C), and motivate your answer with a diagram depicting Sun, Moon and Earth. Select from the options
A: immediately after sunset;
B: during midnight;
C: immediately before sunrise.

**ii)** *(1.2 points)* What is the culmination angle (i.e. the maximal angle above the horizon) of a full moon during winter solstice in Tallinn?

**iii)** *(1.2 points)* The apparent brightness (illuminance at Earth, i.e. luminous flux density) of planets may vary over a wide range of values. By how many times does the apparent brightness of Mars vary between when it's closest and farthest away from Earth?

**iv)** *(1.2 points)* What is the time period on Earth between the moments when Mars is the closest to Earth and farthest?

**v)** *(1.2 points)* From autumn to spring, Venus can never be seen at midnight. For how long maximally it can be seen after the sunset?

**vi)** *(2.5 points)* Express the normalised apparent brightness $I/I_0$ as a function of $R_\oplus$, $R_♀$, and the distance $L$ from Earth to Venus. Here, the normalisation constant $I_0$ can be chosen arbitrarily. *Hint:* $I/I_0$ should be a polynomial of $L^{-1}$.

**vii)** *(1 point)* Find the distance $L = L_0$ when the apparent brightness of Venus is the highest, and the angular distance between Venus and Sun at that moment.

## 9. Magnet in glass (12 points) — *Jaan Kalda, Eero Uustalu.*

**Tools** A transparent cylinder with a cylindrical permanent magnet inside and with foil caps covering its top and bottom; a solid cylinder made from a homogeneous material; a caliper; two boards (for building a slope on which the cylinders can roll down); two bricks (for fixing and adjusting the slope angles); a box for catching rolling cylinders; a ruler — can be also used to release the cylinders; a permanent marker — only for marking on the cylinders, ask an organizer if you need to clean the markings; a pencil — for marking on the boards. **NB! the glass cylinder with the magnet is brittle and expensive, handle it with care and avoid dropping it to the floor.**

In all the tasks, aim to obtain the best possible precision. You will get points both for the steps you take to improve the precision and for the accuracy of your results.

**i)** *(1 point)* Determine the height of the magnet as accurately as possible and estimate the uncertainty of your result.

**ii)** *(3 points)* The acceleration with which a cylinder rolls down a slope depends on the slope angle, and on the ratio $\kappa = I_0/MR^2$, where $I_0$ is its moment of inertia relative to the axis, $M$ — its mass, and $R$ — its radius. Determine the ratio $\kappa$ for the glass cylinder with a magnet inside.

**iii)** *(2.5 points)* Determine the diameter of the magnet. *Hint:* find a way to determine it without knowing the coefficient of refraction of the glass.

**iv)** *(2.5 points)* Notice that the transparent part of the cylinder is actually made of two different materials: the coefficient of refraction of the central part $n_c$ is slightly different from the coefficient of refraction of the outer part of the cylinder $n_o$ (the central part has the same diameter as the permanent magnet). Determine the coefficient of refraction $n_o$ and estimate the uncertainty of the result.

**v)** *(3 points)* Determine the coefficient of refraction $n_c$.