NORDIC-BALTIC PHYSICS OLYMPIAD 2025

1. FLYING DUMBBELL (10 points) — *Jaan Kalda.*

In this problem, we shall study the dynamics of a dumbbell consisting of two steel balls, each with radius $r = 1\,\mathrm{cm}$, connected by a steel rod with diameter $d = 1\,\mathrm{mm}$ and length $l = 10\,\mathrm{cm}$, in the absence of gravity. Unless instructed otherwise, assume steel is perfectly elastic. You may simplify your calculations by assuming $l \gg r$.

i) *(2 points)* Given that the Young's modulus of steel is $Y = 2 \times 10^{11}\,\mathrm{Pa}$ and the density of steel is $\rho = 7800\,\mathrm{kg\,m^{-3}}$, determine the period T of free longitudinal oscillations of the dumbbell. (Do not consider oscillations with standing waves in the rod where the balls remain almost at rest.) Young's modulus is the ratio of a material's stress (force per unit area) to its strain (relative deformation).

ii) *(2 points)* Estimate the impact time τ when a steel ball bounces off a steel wall.

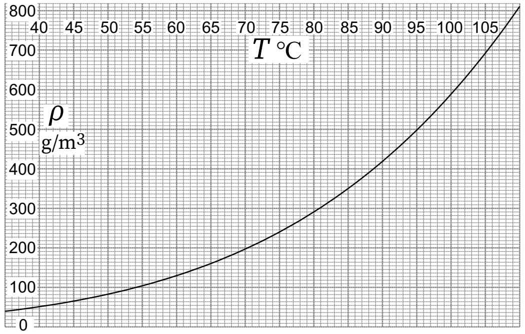
iii) *(2 points)* The dumbbell moves toward a steel wall with velocity $\vec{v} = -v\hat{x}$, with its axis perpendicular to the wall, and bounces back. Here, $\hat{x}$ denotes a unit vector along the axis perpendicular to the wall. Sketch how the x-component v_x of the front ball's velocity (the ball that collides with the wall) depends on time.

iv) *(2 points)* Now, the dumbbell moves toward a steel wall with velocity $\vec{v} = -v\hat{x}$ as before, but its axis forms an angle α with the x-axis. The interaction of the front ball with the wall depends qualitatively on the angle α, with a transition from one type of interaction to another occurring at $\alpha = \alpha_0$. Determine the value of α_0. *Hint:* $\min\left(\frac{\sin x}{x}\right) \approx -0.217$.

v) *(2 points)* Under the assumptions of the previous task, let $\alpha > \alpha_0$. Additionally, assume that while steel is highly elastic, it is not infinitely so: any oscillations excited in the rod will decay by the time the rear ball collides with the wall. Determine the speed with which the centre of mass of the dumbbell departs from the wall.

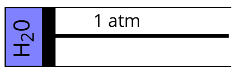
2. EVAPORATION (7 points) — *Jaan Kalda.*

For the subsequent tasks, the graph shows how the density of saturated water vapour in $\mathrm{g\,m^{-3}}$ depends on the temperature in °C.



You may also use the following characteristics of water. Specific heat capacity $c = 4200\,\mathrm{J\,kg^{-1}\,K^{-1}}$; latent heat of vaporization $L = 2260\,\mathrm{kJ\,kg^{-1}}$; density $\rho = 1000\,\mathrm{kg\,m^{-3}}$; molar mass of water $\mu = 18\,\mathrm{g\,mol^{-1}}$. You may also assume water vapour to behave as an ideal gas. The universal gas constant is $R = 8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$.

i) *(2 points)* A cylinder contains a certain amount of water at temperature $T_0 = 90\,^\circ\mathrm{C}$, see the figure. The cross-sectional area of the piston is $S = 1\,\mathrm{dm^2}$. What is the minimum pulling force required to move the piston? The pressure of the surrounding air is $p_0 = 100\,\mathrm{kPa}$.



ii) *(2 points)* If the piston is pulled so that it shifts by $a = 3\,\mathrm{dm}$, the water cools to a temperature of $T_1 = 89\,^\circ\mathrm{C}$; what is the mass of the water under the piston?

Water evaporation has a cooling effect the intensity of which depends on the relative humidity and air convection intensity. It appears, however, that once a dynamical thermal equilibrium is reached, the equilibrium temperature of a wet surface depends only on the relative humidity and the temperature of air and does not depend on the convection speed (as long as the convection is not too weak). This is so because the two competing processes determining the equilibrium state both depend on the thickness of the laminar (non-turbulent) surface layer exactly in the same way. In what follows we shall use the following assumptions. (a) Atop a wet surface (such as a sweating bare skin), there is a layer with a laminar flow of a certain thickness d. (b) Atop the laminar layer, the surrounding turbulent flow keeps a constant temperature T and relative humidity r, both equal to the respective values in the bulk of the surrounding air. (c) Heat flux from beneath the wet surface (e.g. through the skin) can be neglected. (d) The heat conductivity of air $\kappa = 30\,\mathrm{mW\,m^{-1}\,K^{-1}}$ at $T = 70\,^\circ\mathrm{C}$ (neglect the temperature dependence), and the diffusivity of water molecules in air $D = 26\,\mathrm{mm^2\,s^{-1}}$. Neglect the dependence of D on the temperature. Note that the particle flux (net number of molecules passing a cross-section in y–z-plane per second and per cross-sectional area) can be found as $J = D\frac{\mathrm{d}n}{\mathrm{d}x}$, where n denotes the number density (number of molecules per volume).

iii) *(3 points)* Determine the temperature of sweating human skin in a sauna if the air temperature $T = 110\,^\circ\mathrm{C}$ and $r = 3\%$.

3. NUCLEAR REACTORS (6 points) — *Topi Lind.*

In order to maintain a chain reaction in a modern thermal-neutron nuclear reactor one needs three things: 1. nuclear fuel (e.g. U^{235}), 2. moderator (e.g. water) and 3. coolant. In most cases the moderator acts as the coolant as well. Neutrons released from a thermal fission of U^{235} have a mean kinetic energy of approximately $E_0 = 2\,\mathrm{MeV}$. However, neutrons which are that fast are inefficient in triggering fission of U^{235}: neutrons need to be slowed down to an average kinetic energy of $E_f = 0.025\,\mathrm{eV}$. In what follows, justify why non-relativistic approximations can be used unless explicitly instructed otherwise.

i) *(1 point)* The rest energy of neutrons $m_\mathrm{n}c^2 = 940\,\mathrm{MeV}$, the Boltzmann constant $k_\mathrm{B} = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$, and the elementary charge $e = 1.602 \times 10^{-19}\,\mathrm{C}$. What is the required speed of neutrons, i.e. the speed of neutrons with kinetic energy E_f? What is the temperature T_f of a neutron gas where the average kinetic energy of neutrons is E_f?

ii) *(1 point)* What is the initial speed of neutrons, i.e. the speed of neutrons with energy E_0?

iii) *(2.5 points)* From a completely non-relativistic point of view, what should be the mass of the moderator's atoms to slow down the fast neutrons as efficiently as possible? If the mass of the moderator's atoms were to be $M = 135m_\mathrm{n}$, how many collisions with such atoms at a temperature much lower than T_f would a fast neutron need to experience to slow down from E_0 to E_f? Assume that all collisions are elastic and central.

iv) *(1.5 points)* Nuclear fuel, i.e. U^{235}, is placed inside metal rods and pressurized with helium gas to $p_0 = 2.5\,\mathrm{MPa}$. During operation, as U^{235} keeps on fissioning inside the fuel rods, there is a build up of gas inside the rods. With a non-invasive ultrasound measurement we can measure that the gas pressure inside the rod after it is finally picked out from the core is $p = 6.5\,\mathrm{MPa}$. Assuming that the gas released inside the rods is completely made of xenon isotope ${}^{135}_{54}\mathrm{Xe}$ and that the initial gas volume drops from $V_0 = 18\,\mathrm{cm^3}$ to $V = 9\,\mathrm{cm^3}$ due to the swelling of the fuel pellets, how many moles of xenon are released from fission? What is the ratio of helium to xenon inside the rod? The measurements are done at $T_0 = 20\,^\circ\mathrm{C}$; the universal gas constant $R = 8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$.

4. BLACK BOX (12 points) — *Eero Uustalu.*

Tools: a black box with a yellow and a blue lead, two multimeters, a voltage source (that is connected in series with a resistor to limit the maximal current), wires, sheets of graph paper. In the black box, there are three diodes and a resistor.

You are allowed to connect your circuit to the power source only through the yellow lead with a built-in resistor (that is already connected to the power source)!

For each series of measurements, also draw the circuit used!

i) *(4 points)* Determine the electrical circuit inside the black box.

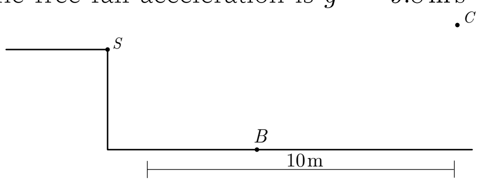
ii) *(2 points)* Determine the resistance of the resistor.

iii) *(6 points)* Determine the opening voltages of all the diodes (at which the current reaches $1\,\mathrm{mA}$).

5. Throwing (6 points) — *Eppu Leinonen.*

i) *(2 points)* A drone starts from the origin at rest and accelerates horizontally with an acceleration g to the $+x$ direction. Simultaneously, a ball is thrown from the point with coordinates $(x,y) = (-h, -h)$. What is the minimum initial speed the ball needs to hit the drone? The free fall acceleration g is antiparallel to the y-axis.

ii) *(4 points)* A stone is thrown from point S (shown in the figure below) with an initial speed v. A boy at point B wishes to hit the stone in midair by throwing a ball simultaneously with the stone's release. He wants to use the minimum possible speed u that will still allow the ball to hit the stone in midair. After calculating the stone's trajectory, he determines the optimal trajectory for the ball and throws it according to his calculations. The collision point C is shown in the figure. Using the scale provided and necessary measurements from the figure, find the initial speeds v of the stone and u of the ball. The free fall acceleration is $g = 9.8\,\mathrm{m\,s^{-2}}$.



6. Birds (4 points) — *Jaan Kalda.*

A long and thin homogenous beam with uniform thickness and square cross-section floats horizontally in water with its top surface parallel to the water surface. A bird lands on one end of the beam, and as a result, the beam sinks so that the edge of the upper face on the bird's side is exactly at the same height as the water surface, while at the other end of the beam the lower face does not rise above the water. What is the maximum number of such birds that this beam can hold above water?

7. Charged rod (6 points) — *Jaan Kalda.* A rod of mass m carries a charge q; both the charge and the mass are homogeneously distributed over its entire length l. The system is in homogeneous magnetic field of strength B, parallel to the z-axis whereas the rod is in the x–y-plane. Neglect any forces except for the Lorentz force. One end of the rod is painted red, and the other — blue.

i) *(2 points)* Consider the case when the rod rotates around its centre of mass. What should be the angular speed ω for the mechanical tension force at the centre of the rod to be zero?

ii) *(4 points)* Consider now a case when initially the blue end of the rod is at the origin ($x = y = 0$), and the red end at $x = l$. The blue end's initial speed is zero while the red end's speed is v, parallel to the y-axis. It turns out that after a certain time t, the red end passes through the origin. Find the smallest possible value for t and express the corresponding value of v in terms of m, q and l.

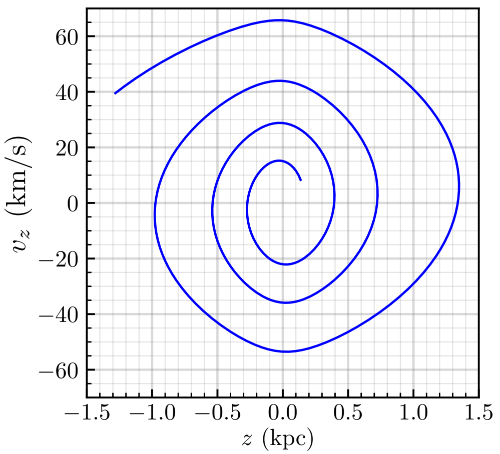
8. Phase spiral (9 points) — *Taavet Kalda.* Here we shall study the motion of Milky Way stars in the Solar neighbourhood in the direction of the z-axis, i. e. perpendicular to the galactic plane. For our purposes, we can model the galactic gravity field as being created by a continuous mass density ρ (that accounts for the masses of stars, dark matter, gas, interstellar dust, etc), and assume that this mass forms an infinite mirror-symmetric plate, i. e. $\rho \equiv \rho(z)$ and $\rho(z) = \rho(-z)$ is independent of x and y. Throughout the problem, you may assume that each star's total energy is conserved over the entire considered time period. Gravitational constant $G = 6.67\times10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} = 4.30\times10^{-3}\,\mathrm{pc\,M_\odot^{-1}\,(km/s)^2}$.

i) *(1 point)* Assuming that the mass density is constant over the plate's thickness. i.e. $\rho(z) = \rho_0$, what is the acceleration a_z of a star at a distance z from the mid-plane?

ii) *(0.5 points)* Consider a star that starts with zero velocity at a distance of $z = a$ from the mid-plane. With what period does it start oscillating around the mid-plane?

In reality, density decreases with growing $|z|$. Measuring density has been a great challenge because of contributions from dark and other difficult-to-see matter. Here, we consider a breakthrough method of doing it. Consider the distributions of the stars in our neighbourhood on the z–v_z phase plane, where each star is a dot with coordinates (v_z,z); v_z denotes the z-component the star's velocity, and z – the vertical coordinate. Initially, these dots were distributed nearly homogeneously, but some time ago, the Milky Way was perturbed externally, probably by a passing-by dwarf galaxy; this shuffled the positions and velocities of stars, creating a bar-shaped overdensity region. When moving within that bar-shaped region from the centre to the periphery, the total energy per mass of stars increased monotonously. Over time, this overdensity region started "winding up", due to the oscillation periods of stars in the vertical plane depending on their oscillation amplitude z_m, and evolved into a spiral pictured below (Antoja et al. 2018, Nature 561, 360). An observation that you need to exploit below is that the ordering of stars by energies along the spiral today remains the same as it was at the time of perturbation.

The oscillation period of stars depends on the amplitude z_m because the gravitational potential (the potential energy per mass) $\Phi(z)$ is not parabolic. In such a case, the period can be approximately found by substituting the real $\Phi(z)$ with a kz^2 matching $\Phi(z)$ at $z = z_m$, i. e. with $k = \Phi(z_m)/z_m^2$.



iii) *(2.5 points)* At the intersection points of the spiral with $v_z = 0$, calculate $\Phi(z)$ by interpolating data linearly where appropriate; plot your results (this follows the analysis of Guo et al. 2024, ApJ, 960, 133).

iv) *(1 point)* Assuming that the mass density is almost constant for $|z| \le 0.3\,\mathrm{kpc}$, what is the mass density near $z = 0$?

v) *(2 points)* Dark matter is an "invisible" form of matter that only interacts by gravity. In general, it is found that dark matter forms halos that extend significantly farther than visible matter structures. By assuming that the dark matter density doesn't vary significantly within the volume of interest and that it starts dominating far away from the galactic plane, from around $z = 0.7\,\mathrm{kpc}$, estimate the local dark matter density ρ_{DM}.

vi) *(2 points)* How long ago did the perturbation occur?

9. Hot plate (12 points) — *Jaan Kalda.* *Tools:* a resistor with resistance $R = 220\,\Omega$ mounted in one corner of a foam plastic plate (resistor's upper surface is painted black), a power supply (on the power supply, readings other than voltage are not reliable!), two identical $40\,\mathrm{mm}\times40\,\mathrm{mm}$ aluminium plates that differ only in their coating: one is polished, the other is anodized black and can be assumed to be a black body, 4 silicone rubber pads of thickness $t = 0.8\,\mathrm{mm}$, a foam cup with hot water, tweezers, a stopwatch, an infrared thermometer (press the knob twice and take a reading) , sheets of graph paper and paper tissues for cleaning. Note that for thermal radiation, water can be also assumed to be a black body.

Do not apply voltages higher than $U = 15\,\mathrm{V}$ to the resistor as this will lead to a melting of the foam plastic! Please handle foam cups with care and do not break them (you will not get a spare one)!

i) *(3 points)* Determine the emissivity (thermal radiation power relative to a black body) of the polished aluminium plate. *Hint*: infrared thermometers measure temperature via thermal radiation power (assume it is calibrated for objects of 100% emissivity); within relevant temperature ranges, this relationship can be approximated as linear,

ii) *(3 points)* Determine the heat transfer coefficient between the top surface of the black aluminium plate and air (i. e. the coefficient of proportionality between heat dissipation power and the temperature difference) that includes both conductive and radiative heat transfer.

iii) *(2 points)* Determine the heat capacity of a metal plate.

iv) *(4 points)* Determine the heat conductance of silicon rubber pads (neglect their heat capacitance).