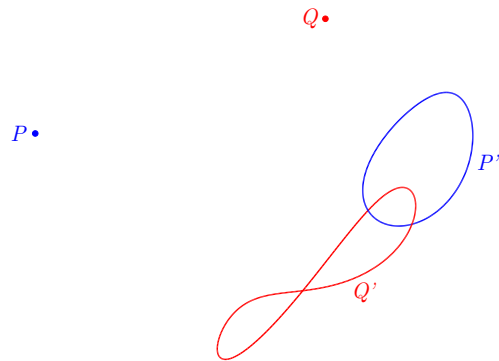


1. MOVING LENS (6 points) — *Eppu Leinonen*. The centre of a thin converging lens moves along a circle while the orientation of the lens remains fixed; the optical axis of the lens lies in the plane of the circle (the plane of the figure). Two fixed points P and Q , also in this plane, are imaged by the lens; the images are always real. As the lens moves, each image traces a closed curve in the plane of the figure: $P \rightarrow P'$, $Q \rightarrow Q'$. The two points and both curves are shown in the figure.



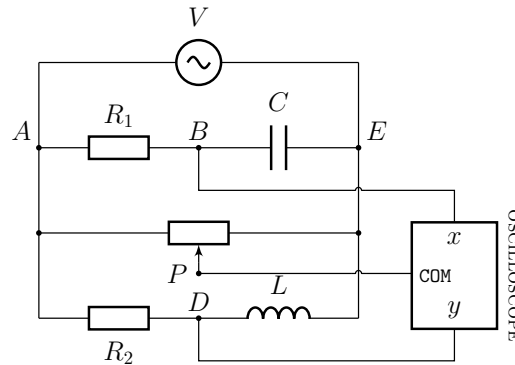
i) (2 points) Construct the circle along which the lens centre moves.

ii) (4 points) Determine the direction of the lens plane, i. e. construct a line parallel to the lens.

In the construction tasks, draw the construction on the provided sheet with the figure, detail the steps of your construction, and explain why your construction works.

2. LISSAJOUS BRIDGE (6 points) — *Jaan Kalda*. A circuit has four nodes A, B, E, D . An AC source of amplitude V is applied between A and E . A resistor R_1 connects A to B and a capacitor C connects B to E ; a resistor R_2 connects A to D and an inductor L connects D to E . A potentiometer with a sliding contact P is connected between A and E . The voltage between P and B feeds the x -input of an oscilloscope, and the voltage between P and D feeds the y -input; both input channels have the same gain. The oscilloscope plots the voltage V_x from the x -input and the voltage V_y from the y -input in the V_x - V_y -plane. The position of the sliding contact P is

adjusted until the figure on the screen degenerates into a line segment. It appears that in that case, the line segment makes an angle α with the x -axis, and the sliding contact P divides the potentiometer's length in the ratio $1 : 2$ from left to right in the figure. Find the amplitude of the voltage between B and P .



3. KIRILL ON A SWING (8 points) — *Kaarel Hänni*. A swing has rigid rods attaching it to a horizontal pivot, so it can rotate freely in a vertical plane. Initially the swing hangs vertically below the pivot with Kirill standing upright on the seat; his friend gives him an initial angular velocity ω_0 about the pivot. Kirill is practising athletic swinging: whenever the swing momentarily has zero angular velocity, he quickly squats; whenever the rods are again vertical, he quickly stands up. Treat Kirill as a point mass at distance a from the pivot when standing and b when squatting ($a < b$); squatting and standing are motions along the rods. Neglect friction, air resistance, and the mass of the swing. Gravitational acceleration is g .

i) (1.5 points) Sketch, qualitatively, how Kirill's angular speed depends on time during the first full period of the swing.

ii) (2 points) On the phase diagram (angular velocity vs. angle), sketch, qualitatively, the trajectory from the initial push until the swing first goes over the top. The total number of periods shown need not be correct.

iii) (4.5 points) Find how many times Kirill must stand up before the swing first goes over the top. Evaluate your answer for $a = 2.5$ m, $b = 3.0$ m, $\omega_0 = 1.0$ rad/s, and $g = 10$ m/s².

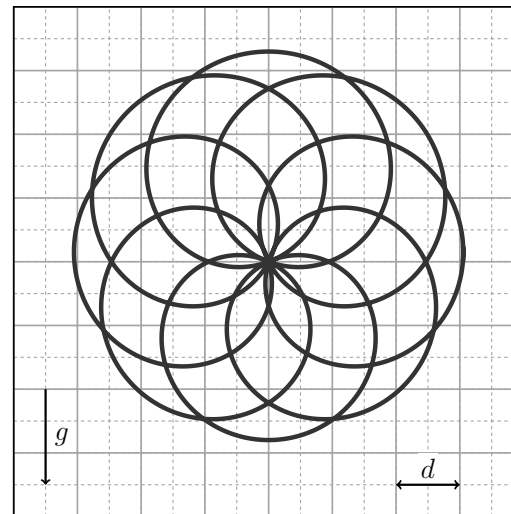
4. ROD AND BEAD (8 points) — *Eero Ristolainen, Jaan Kalda*. A bead is threaded on a rigid frictionless rod. The rod rotates with constant angular velocity ω about a horizontal axis that is perpendicular to the rod and passes through a fixed point on it. Gravitational acceleration is g .

i) (1 point) With suitable initial conditions, the trajectory of the bead is a circle. Sketch this trajectory in the plane perpendicular to the rotation axis, in a coordinate system with the axis at the origin.

ii) (1 point) Given that the trajectory is exactly a circle, find its radius.

iii) (2 points) The circular trajectory of the previous tasks is unstable. To stabilise it, one attaches a spring that produces a restoring force $-kx$ when the bead is displaced by x along the rod from the axis of rotation. For what values of the stiffness k can the bead move along a stable circular trajectory, and what is the radius of such a trajectory?

iv) (4 points) Next, the spring is removed, the bead is given a charge q , and a uniform electric field of strength E whose vector rotates with angular velocity Ω in the same direction as the rod is introduced. With suitable initial conditions, the bead follows the trajectory shown below. Find Ω , g , and the ratio $a_E = Eq/m$, where m is the mass of the bead, in terms of ω and d , the side length of the grid squares shown in the figure. You may take measurements from the figure.



5. SATURATION PRESSURE OF WATER (8 points) — *Eero Uustalu*. Tools: transparent plastic tube (open at both ends, Luer lock to connect with a syringe at one end, length ~ 1.5 m, inner diameter ~ 1.5 mm); plastic syringe with a water-tight Luer fitting to the tube; small open container with a few millilitres of dodecane (labelled "D"); metal pin with outer diameter ~ 2 mm to close the tube tightly (if it gets stuck, ask the invigilator to help remove it); measuring tape; container with water (labelled "W"); a large container for waste liquids; a small wooden stick; a few strips of masking tape; paper towel.

Water and dodecane do not dissolve in each other. The density of dodecane is less than the density of water.

Avoid skin and eye contact with dodecane, and do not ingest it; wash your hands after handling.

During the experiment, the tube can only be filled with dodecane: **the inner surface of the tube must never touch water!**

If necessary, you can have one replacement tube, one replacement syringe, and reasonable additional amounts of dodecane and water.

Determine the saturated vapour pressure p_{sat} of water at room temperature; room temperature and air pressure are recorded by the invigilator at the start of the session: $T_{\text{room}} =$ _____ °C; $p_{\text{air}} =$ _____ kPa.

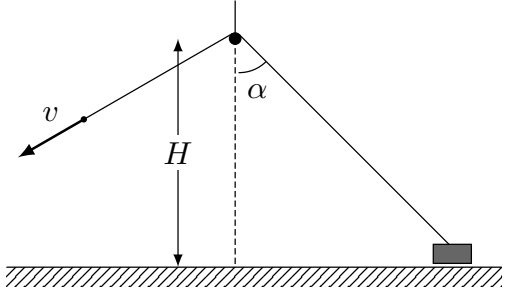
Note that at typical room temperatures, the saturated vapour pressure of dodecane is much lower than that of water.

i) (2 points) Describe your measurement procedure and explain the physical principle it is based on.

ii) (4 points) Perform the measurements. Tabulate all quantities you measure and document the sequence of operations.

iii) (2 points) Determine p_{sat} at room temperature. What is the dominant source of systematic error?

6. PULL-UP BAR ROPE (5 points) — *Jaan Kalda*. A rope is thrown over a horizontal pull-up bar at height H above the frictionless floor. One end of the rope is tied to a heavy weight resting on the floor; the other end is pulled so that the length of the rope between the bar and the weight decreases at a constant rate v . At a given moment, this segment makes an angle α with the vertical (see figure); the weight is still in contact with the floor. Gravitational acceleration is g .



- i)** (1 point) Find the speed u of the weight.
ii) (1 point) Find the magnitude of the acceleration of the weight.
iii) (3 points) Find the angle α_0 at which the weight lifts off the floor.

7. WATER HOSE (5 points) — *Ralf Robert Paabo*. A liquid with density ρ flows in a pipe with diameter D . At low mean speeds v , the flow is laminar (uniform and without vortices); the drag force per unit area $F_L = \mu du/dr$ on the liquid comes from wall friction, where μ (unit Pa · s) is the dynamic viscosity, u is the local speed, and r is the distance from the axis. When v is large enough, the flow becomes turbulent, filled with vortices whose velocity fluctuations are of the order of v itself. The drag F_T still comes from wall friction $\mu du/dr$, but the vortices squeeze the near-wall layer (across which the flow speed drops from v to zero) to a thickness $\delta \ll D$ where the drag force per unit area $\mu v/\delta$ is of the order of the dynamic pressure ρv^2 carried by the vortices.

i) (1.5 points) At any given flow speed, only one of the two mechanisms actually dominates the drag – but the dimensionless ratio of the two characteristic drag scales, $R = F_T/F_L$, can be computed regardless and tells us which regime we are in. This ratio is the *Reynolds number*. Express R as a product of

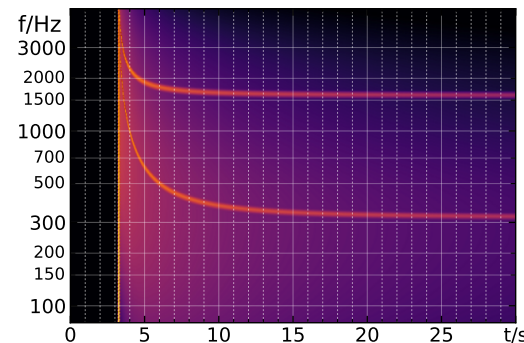
powers of ρ , v , D , and μ .

A water pump with output power $P = 250$ W is used for watering a garden; it draws water from a depth $h = 20$ m directly into a hose of length $s = 40$ m and inner diameter $d = 13$ mm. Water leaves the end of the hose at a volumetric rate $Q = 25$ L · min⁻¹. Density of water is $\rho_w = 1000$ kg · m⁻³, viscosity $\mu_w = 1.1 \times 10^{-3}$ Pa · s and gravitational acceleration is $g = 9.8$ N · kg⁻¹.

ii) (0.5 points) Determine the flow type in the hose. It is known that flow is turbulent when $R > 2500$ and laminar otherwise.

iii) (3 points) The gardener is cleaning tools by directing the water jet onto dirty surfaces. Where the jet meets the surface, the pressure rises above atmospheric; this excess pressure is what removes dirt. To make cleaning more efficient, the gardener attaches a spray nozzle set to narrow-jet mode, so that the exit cross-section of the nozzle is $f = 15\%$ of the hose's cross-section. By what factor does the volumetric flow rate Q change? By what factor does the excess pressure at the dirty surface change? Find both within 1% accuracy. You are allowed to use numerical methods.

8. JET SOUND (8 points) — *Teo Kai Wen, Jaan Kalda*. A fighter jet flies past a ground observer along a straight horizontal line at constant supersonic velocity, passing at closest-approach distance d . The jet's Mach number is $M = v/c > 1$, where v is the speed of the jet and c is the speed of sound. The air is at rest. The observer records the sound as a spectrogram (intensity colour-coded as a function of frequency and time; black means silence), shown below.

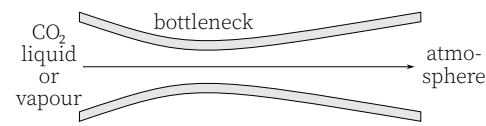


i) (2 points) Explain, qualitatively, the features seen in the spectrogram.

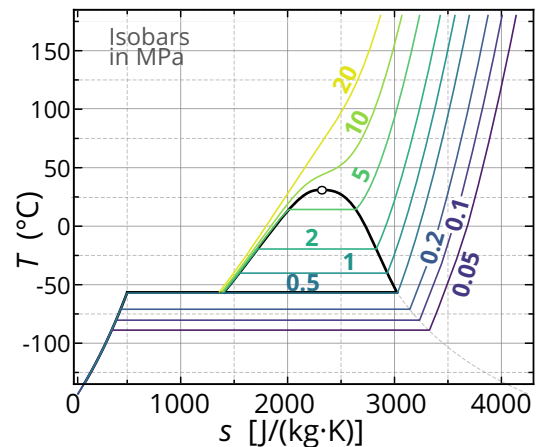
ii) (3 points) From the spectrogram, find the Mach number M .

iii) (3 points) Find the closest-approach distance d given that $c = 340$ m · s⁻¹.

9. CO₂ FIRE EXTINGUISHER (8 points) — *Jaan Kalda*. A fire extinguisher contains liquid CO₂ in equilibrium with its saturated vapour at room temperature $T_0 = 298$ K. Consider two scenarios: **(a)** the container is held upside-down so that the liquid phase flows to the nozzle; **(b)** it is held upright so that the saturated vapour flows to the nozzle. The nozzle has the shape of a converging-diverging channel (see figure), and the flow through it can be modelled as reversible and adiabatic. Atmospheric pressure is $p_{\text{atm}} = 1.0 \times 10^5$ Pa; the CO₂ triple point is at $(T_t, p_t) = (216.6$ K, 0.518 MPa).



The temperature-entropy diagram of CO₂ with isobars is provided below.



i) (1.5 points) Identify the phase or phases present and their temperature in the stream emerging from the nozzle.

ii) (3.5 points) Find the mass fraction x of solid CO₂ in the stream for both scenarios.

iii) (3 points) Now consider an arbitrary pure substance in equilibrium with its sat-

urated vapour at temperature T , and suppose that only the vapour (not the liquid) escapes through a nozzle, undergoing reversible adiabatic expansion. Under what condition on the vaporisation latent heat L (per unit mass), the isobaric specific heat of the vapour c_p , and the temperature T does a fraction of the escaping vapour condense into droplets, even for a vanishingly small pressure drop? Does condensation occur for water vapour at $T = 373$ K ($L \approx 2260$ kJ · kg⁻¹, $c_p \approx 2.0$ kJ · kg⁻¹ · K⁻¹)?

10. BALL MAGNET (10 points) — *Jaan Kalda*. *Tools:* ball-shaped neodymium magnet of diameter $d = 10$ mm with remanence $B_r = 1.2$ T, density $\rho_M \approx 7500$ kg · m⁻³; stand; permanent marker; ruler; measuring tape; flat iron disk; square wooden plate with an iron pin inserted at one side; flat wooden plate with guiding rails and non-slipping surface between them; piece of titanium of diameter 2.4 mm and length 3.4 mm fixed to a string. Density of titanium $\rho_{\text{Ti}} \approx 4500$ kg · m⁻³.

A uniformly magnetised sphere produces an external field identical to that of a point magnetic dipole with a moment $\mu = \frac{4}{3}\pi R^3 B_r/\mu_0$, where R is the sphere's radius. Along the axis of a dipole, $\vec{B} = \mu_0 \vec{\mu}/(2\pi r^3)$, where r is the distance from the dipole. The dipole moment of the Earth is pointing southwards.

Warning: measure as far as possible from iron objects (table frames, chairs), which strongly perturb the geomagnetic field.

i) (2 points) Find the direction of the magnetisation of the magnet: mark on its surface a dot at the point where the straight magnetic field line exits, and a cross where it enters. *Before leaving the room, give the magnet with the markings to the invigilator.*

ii) (4 points) Find the magnitude of the Earth's magnetic field $|\vec{B}_E|$ at the laboratory location.

iii) (4 points) Titanium is a paramagnetic material with magnetic susceptibility $\chi \ll 1$. A small sample of volume V in an inhomogeneous field $\vec{B}(\vec{r})$ experiences a force $\vec{F} = \frac{1}{2}(\chi V/\mu_0) \text{grad } B^2$ ('grad' denotes derivative in the direction of greatest change). Find χ for the supplied titanium wire piece.